

Unit 6

Tool Kit: Writing Proofs

Standards

The student will:

- 1.3 Solve problems related to angles and parallel lines.
- 2.1 Solve problems involving interior and exterior angles of triangles.
- 2.2 Use properties related to altitudes and medians of triangles to solve problems.
- 2.3 Use properties related to isosceles triangles to solve problems.
- 7.1 Use the Pythagorean Theorem and its converse to solve problems.
- 10.1 Recognize the hypothesis and conclusion of an if-then statement and state its converse.
- 10.2 Use inductive reasoning to observe data, recognize patterns, and make generalizations.
- 10.3 Complete geometric proofs by applying appropriate postulates and theorems.

Day 1
Tool Kit Practice and Arrow Diagrams

Connection

Prior Work	Current Big Idea	Future Work
<ul style="list-style-type: none"> Fundamental terms and properties relating to angles. 	<ul style="list-style-type: none"> Write conditional statements. Write justifications to for solutions to given problems. 	<ul style="list-style-type: none"> Write proofs for given geometric relations.

Lesson Process

Steps	Student Activity	Teacher Support	Comment/Evaluation
<p>Launch</p> <p>10-15 minutes</p>	<ul style="list-style-type: none"> Read Introduction, TK-1 and TK-2. Follow teacher in completing TK-2. 	<ul style="list-style-type: none"> Demonstrate the use of justifying each step of the solution to a given problem by using a know property. 	<ul style="list-style-type: none"> Students need to use their own language as they learn new concepts.
<p>Explore</p> <p>20-25 minutes</p>	<ul style="list-style-type: none"> Complete TK-3 through TK-6 with study team and teacher. 	<ul style="list-style-type: none"> In TK-3, expect to see an equation such as $2x + 3 + 5x + 9 = 180$. Students should write and phrase or a sentence justifying the above equation by a statement such as: "Two angles that form a straight angle are supplementary." Facilitate the writing of complete reasons rather than compacted 	<ul style="list-style-type: none"> There are more than one method in solving each of the problems. Be sure to acknowledge all correct answers.

Steps	Student Activity	Teacher Support	Comment/Evaluation
		versions such as: <ul style="list-style-type: none"> • “$\Delta 180^\circ$” rather than “the sum of the measures of interior angles of a triangle is 180.” 	
Summarize 8-10 minutes	<ul style="list-style-type: none"> • Demonstrate and discuss TK-3 through TK-6. • Add TK-7 in the tool kit. 	<ul style="list-style-type: none"> • Stress the importance for the justifications. 	
Homework	11 TK-8 through TK-13.		

Day 2
Proofs of Familiar Ideas

Connection

Prior Work	Current Big Idea	Future Work
<ul style="list-style-type: none"> Justify given step using properties and definitions relating to angles and lines. 	<ul style="list-style-type: none"> Write justifications using facts rather than opinions. 	<ul style="list-style-type: none"> Complete formal proofs for given statements relating to geometric figures.

Lesson Process

Steps	Student Activity	Teacher Support	Comment/Evaluation
<p>Launch</p> <p>8-10 minutes</p>	<ul style="list-style-type: none"> Complete TK-14 through TK-15 with teachers. 	<ul style="list-style-type: none"> The emphasis is to have students consider the difference between what they believe to be true and what they can prove is true. Stress the issues of opinion versus fact. 	
<p>Explore</p> <p>20-25 minutes</p>	<ul style="list-style-type: none"> Complete TK-15 through TK-17 with study team. 	<ul style="list-style-type: none"> Capitalize on the logical, step-by-step mathematical process the students have used previously. Students may want to start on TK-18 through TK-22. 	<ul style="list-style-type: none"> Be more concerned about the logic of the students' statements rather than formulized statements relating to the properties.
<p>Summarize</p> <p>8-10 minutes</p>	<ul style="list-style-type: none"> Share discussion on TK-16. Add TK-17 to the tool kit. 	<ul style="list-style-type: none"> From TK-15 through tK-17, emphasize that a proof convinces an audience that a conjecture is true for ALL cases 	

Steps	Student Activity	Teacher Support	Comment/Evaluation
		that fit the conditions of the conjecture.	
Homework	<ul style="list-style-type: none">TK-18 through TK-22.		

Day 3

Theorems and More Proofs of Familiar Ideas

Connection

Prior Work	Current Big Idea	Future Work
<ul style="list-style-type: none">Justify given step using properties and definitions relating to angles and lines.	<ul style="list-style-type: none">Write justifications using facts rather than opinions.	<ul style="list-style-type: none">Complete formal proofs for given statements relating to geometric figures.

Lesson Process

Steps	Student Activity	Teacher Support	Comment/Evaluation
Launch 8-10 minutes	<ul style="list-style-type: none">Read TK-25.Complete for addition to tool kit.	<ul style="list-style-type: none">Conjectures are what we believe to be true.Theorems are proven for all cases that meet the given condition(s).	
Explore 20-25 minutes	<ul style="list-style-type: none">Complete TK-26 through TK-29 with study team.	<ul style="list-style-type: none">The logical steps and justifications are stressed in solving familiar problems.	<ul style="list-style-type: none">Students need to complete TK-26 through TK-29 in class with guidance and monitoring from teacher.
Summarize 8-10 minutes	<ul style="list-style-type: none">Demonstrate Tk-26 through TK-29.	<ul style="list-style-type: none">Use student proofs as models to be evaluated by other team members.	<ul style="list-style-type: none">The formality of the statements are not as important as the logic and facts used in the justifications or explanations for the given

Steps	Student Activity	Teacher Support	Comment/Evaluation
			problems.
Homework	<ul style="list-style-type: none"> • TK-30 through TK-35. 	<ul style="list-style-type: none"> • TK-33 through TK-36 are Algebra I review problems. 	

Day 4
Flowchart Proofs

Connection

Prior Work	Current Big Idea	Future Work
<ul style="list-style-type: none"> Justify given step using properties and definitions relating to angles and lines. 	<ul style="list-style-type: none"> Use flowcharts to show the steps in solving problems. 	<ul style="list-style-type: none"> Complete formal proofs for given statements relating to geometric figures.

Lesson Process

Steps	Student Activity	Teacher Support	Comment/Evaluation
<p>Launch</p> <p>8-10 minutes</p>	<ul style="list-style-type: none"> Complete TK-37 with teachers. 	<ul style="list-style-type: none"> Flowcharts provide another way to show the steps in solving problems. No formality of the flowchart symbols. Just the follow of the logic in solving the problem. 	
<p>Explore</p> <p>20-25 minutes</p>	<ul style="list-style-type: none"> Complete TK-38 through TK-42 with study team. 	<ul style="list-style-type: none"> Emphasize following logical steps and use of facts rather than opinions in the explanation or justifications for the steps in solving given problems. 	<ul style="list-style-type: none"> Flowcharting is one method of helping students show steps in solving problems in a logical manner.
<p>Summarize</p> <p>8-10 minutes</p>	<ul style="list-style-type: none"> Share demonstrations for TK-38 through tK-42. 	<ul style="list-style-type: none"> The steps each study team uses will be different. The logic and the facts used in the explanations are the important factors. 	

Steps	Student Activity	Teacher Support	Comment/Evaluation
Homework	<ul style="list-style-type: none"><li data-bbox="394 337 709 370">• TK-43 through TK-47.		

Day 5
Flowchart Proofs of parallelism Conjectures

Connection

Prior Work	Current Big Idea	Future Work
<ul style="list-style-type: none"> Justify given step using properties and definitions relating to angles and lines. 	<ul style="list-style-type: none"> Use flowcharts to show the steps in solving problems. 	<ul style="list-style-type: none"> Complete formal proofs for given statements relating to geometric figures.

Lesson Process

Steps	Student Activity	Teacher Support	Comment/Evaluation
Launch 8-10 minutes	<ul style="list-style-type: none"> Read TK-48 and TK-49. Add TK-49 to tool kit. 	<ul style="list-style-type: none"> Talk about postulates or axioms as being accepted facts or basic truth. 	
Explore 20-25 minutes	<ul style="list-style-type: none"> Complete TK-50 through TK-55 with study team. 	<ul style="list-style-type: none"> Monitor student work for logical reasoning in the steps in solving given problems. 	
Summarize 8-10 minutes	<ul style="list-style-type: none"> Share demonstrations for TK-50 through TK-55. 	<ul style="list-style-type: none"> Discuss the different steps used in solving the problems. 	
Homework	<ul style="list-style-type: none"> TK-56 through TK-62. 		

Day 6
Proving the Pythagorean Theorem

Connection

Prior Work	Current Big Idea	Future Work
<ul style="list-style-type: none"> Construct flowcharts to show steps in solving problems. 	<ul style="list-style-type: none"> Prove the Pythagorean Theorem. 	<ul style="list-style-type: none"> Construct proofs for given mathematical statements.

Lesson Process

Steps	Student Activity	Teacher Support	Comment/Evaluation
<p>Launch</p> <p>8-10 minutes</p>	<ul style="list-style-type: none"> Complete TK-62 with teacher. 	<ul style="list-style-type: none"> Complete TK-62 with using flowchart to show steps and justifications. 	
<p>Explore</p> <p>20-25 minutes</p>	<ul style="list-style-type: none"> Complete TK-63 through TK-66 with study team. 	<ul style="list-style-type: none"> The proof for the Pythagorean Theorem combines geometric dissection together with algebra. 	<ul style="list-style-type: none"> Good time to demonstrate the many other interesting proofs for the Pythagorean Theorem.
<p>Summarize</p> <p>8-10 minutes</p>	<ul style="list-style-type: none"> Demonstrate the proof for the Pythagorean Theorem. 	<ul style="list-style-type: none"> Discuss the various methods used in proving the Pythagorean Theorem. 	
<p>Homework</p>	<ul style="list-style-type: none"> TK-67 through TK-72. 		

Day 7
Converses and Counterexamples

Connection

Prior Work	Current Big Idea	Future Work
<ul style="list-style-type: none"> Complete proofs using flowcharts. 	<ul style="list-style-type: none"> Form converses for given conditional statements. Show counterexamples for given generalizations. 	<ul style="list-style-type: none"> Construct indirect proofs.

Lesson Process

Steps	Student Activity	Teacher Support	Comment/Evaluation
<p>Launch</p> <p>10-15 minutes</p>	<ul style="list-style-type: none"> Complete TK-73 and TK-74 with teacher. Add TK-73 and TK-74 to the tool kit.. 	<ul style="list-style-type: none"> A converse of a conditional statement is formed by switching the if and then parts of the statement. Discuss examples in which the statement and its converse are both true, both false, or one true and one false. Have students find counterexamples for given statements. 	<ul style="list-style-type: none"> In a converse, the condition becomes the conclusion and the conclusion becomes the condition. One counterexample is enough to disprove a generalization.
<p>Explore</p> <p>15-20 minutes</p>	<ul style="list-style-type: none"> Complete TK-75 through TK-79 with study team. 	<ul style="list-style-type: none"> TK-75 may be used to show that converses to given statements are not always true. A counterexample may be used to 	

Steps	Student Activity	Teacher Support	Comment/Evaluation
		show that the statement is false.	
Summarize 8-10 minutes	<ul style="list-style-type: none"> • Demonstration of TK-75 through TK-79, 	Demonstrate the importance of using counterexample to show that some generalizations are false.	
Homework	<ul style="list-style-type: none"> • TK-80 through TK-86. 		

Day 8 Two Column Proofs

Connection

Prior Work	Current Big Idea	Future Work
<ul style="list-style-type: none"> Use flowchart to construct proofs for given statements. 	<ul style="list-style-type: none"> Construct two column proofs. 	<ul style="list-style-type: none"> Complete proofs for given mathematical statements.

Lesson Process

Steps	Student Activity	Teacher Support	Comment/Evaluation
Launch 10-15 minutes	<ul style="list-style-type: none"> Complete TK-87 with teacher. 	<ul style="list-style-type: none"> Introduction of two column proofs. The third statements in the proof for TK-87, the reason stated is “same segment.” The use of “reflexive property” would be a more appropriate choice. The reason for the last statement should be “corresponding parts of congruent triangles are congruent.” 	<ul style="list-style-type: none"> Writing the reasons in words rather than using abbreviated symbols help student solidify the logical development of each step of the proof. The statements, not the abbreviations are more universally use and understood.
Explore 20-25 minutes	<ul style="list-style-type: none"> Complete TK-88 through TK-82 with study team. 	<ul style="list-style-type: none"> Monitor and facilitate the completion of two column proofs. Talk about the utility of extra lines and the advantage of focusing on just part of the diagram in general. Talk about marking diagram to show 	

Steps	Student Activity	Teacher Support	Comment/Evaluation
		congruence of parts.	
Summarize 8-10 minutes	<ul style="list-style-type: none"> • Demonstrate TK-90. 	<ul style="list-style-type: none"> • Select study team for demonstration of TK-90. 	
Homework	<ul style="list-style-type: none"> • TK-93 through TK-98. 		

Day 9
Proof by Contradiction

Connection

Prior Work	Current Big Idea	Future Work
<ul style="list-style-type: none"> Find counterexamples for given generalizations. 	<ul style="list-style-type: none"> Complete indirect proofs. 	<ul style="list-style-type: none"> Complete proofs for given mathematical statements.

Lesson Process

Steps	Student Activity	Teacher Support	Comment/Evaluation
Launch 8-10 minutes	<ul style="list-style-type: none"> Complete TK-99 and TK-100 with teacher. 	<ul style="list-style-type: none"> Complete examples of indirect proofs. The indirect proof begins by assuming that something is true and then showing that such an assumption eventually leads to a contradiction of a know fact. 	
Explore 20-25 minutes	<ul style="list-style-type: none"> Complete TK-100 through TK-105 with study team. 	<ul style="list-style-type: none"> Complete TK-100 through TK-105 as a class. 	<ul style="list-style-type: none"> Student are not familiar or comfortable with the format of using indirect proofs.
Summarize 8-10 minutes	<ul style="list-style-type: none"> Start TK-107 and TK-108. Add TK-100 to the tool kit. 	<ul style="list-style-type: none"> If TK-100 through TK105 is completed together as a class, students should start on TK-107 and TK-108. Discuss TK-108. 	

Steps	Student Activity	Teacher Support	Comment/Evaluation
Homework	<ul style="list-style-type: none">TK-106 through TK-110.		<ul style="list-style-type: none">Good review problems.

Day 10
Unit and Tool Kit Review

Connection

Prior Work	Current Big Idea	Future Work
<ul style="list-style-type: none"> Complete proofs by using flowcharts, two columns, or contradiction. 	<ul style="list-style-type: none"> Review of properties for algebra and geometry. Review of construction of proofs. 	<ul style="list-style-type: none"> Construction of algebraic and geometric proofs.

Lesson Process

Steps	Student Activity	Teacher Support	Comment/Evaluation
<p>Launch</p> <p>8-10 minutes</p>	<ul style="list-style-type: none"> Review concepts and skills from the tool kit. Prepare for tool kit clean-up. 	<ul style="list-style-type: none"> Discuss the use of the tool kit. Prepare for tool kit clean-up. 	
<p>Explore</p> <p>20-25 minutes</p>	<ul style="list-style-type: none"> Complete TK-111 through TK-128 with study team. 	<ul style="list-style-type: none"> Selected problems may be assigned to different teams. Prepare teams to share assigned problems for review. TK-127 and TK-128 are more ways of proving the Pythagorean Theorem. TK-123 through TK126 extends students' earlier investigation of triangle congruence properties to quadrilaterals. 	<ul style="list-style-type: none"> TK-123 through TK-126 may be used for a mini project.

Steps	Student Activity	Teacher Support	Comment/Evaluation
Summarize 8-10 minutes	<ul style="list-style-type: none">• Share assigned problems for review.		
Homework	<ul style="list-style-type: none">• Complete unfinished problem from TK-111 through TK-128.		