

Unit 5

Estimating Fish Population: Numerical, Geometric, and Algebraic Ratios

Standards

- 7.6 Determine the slope of a line when given two points on a line or an equation of the line.
- 9.4 Solve equations involving proportions.

Day 1 Exploring Ratios

Connections

Prior Work	Current Big Idea	Future Work
<ul style="list-style-type: none"> Operations with integers. Ratio as equivalent fractions. 	<ul style="list-style-type: none"> Ratio between numbers. Ratio from a geometric perspective - enlarging and reducing simple lattice polygons. 	<ul style="list-style-type: none"> Solve equations involving proportions.

Lesson Process

Steps	Student Activity	Teacher Support	Comment/Evaluation
<p>Launch</p> <p>8-15 minutes</p>	<ul style="list-style-type: none"> Read theme problem EF-0. 	<ul style="list-style-type: none"> Have students discuss theme problem EF-0 but do not dwell on specifics. Before starting EF-1 through eF-5 try example such as “The High School Science Club went to visit the Marine Science Center. Four teachers, fourteen boys and seventeen girls made the trip. What was the ratio of teacher to girls? What was the ratio of boys to teachers?” $[4:17 \text{ or } \frac{4}{17}] [14:4 \text{ or } \frac{14}{4}]$ Emphasize the order matters. 	<ul style="list-style-type: none"> The notion of “ratio” is one of the fundamental concepts of mathematics. Start with a review of numerical ratios. Give or solicit examples to show how extensively ratios are used: in sports, in science, in economics, etc. This unit examines the idea of “ratio” in several contexts: numeric, graphic, geometric, and algebraic. Ratio as comparison of numbers by division.

Steps	Student Activity	Teacher Support	Comment/Evaluation
	<ul style="list-style-type: none"> Complete EF-1 through EF-5. 	<ul style="list-style-type: none"> Complete EF-1 through EF-5 with class. All information within each problem is not given in the same units, students sometimes need to complete a subproblem first. 	<ul style="list-style-type: none"> In order to emphasize the comparison involved in ratios, students should label units to indicate what is being compared until they establish that the items are in the same units. Emphasize the importance of order and identifying the variables being compared.
<p>Explore 25-30 minutes</p>	<ul style="list-style-type: none"> Complete EF-6 through EF-8 with study team. 	<ul style="list-style-type: none"> Complete EF-6 through EF-8. Make sure students understand EF-7 before they try EF-8. Check the teams' diagrams as you circulate. Watch for students who compute perimeters by counting dots rather than segments. 	<ul style="list-style-type: none"> For convenience, the ratios in this text, will be listed in the order that compares "new" to "original."
<p>Summarize 10-15 minutes</p>	<ul style="list-style-type: none"> Complete EF-9. Share discussion with class. 	<ul style="list-style-type: none"> "What observations can you make about the ratio of the sides and perimeters? If so, what?" "What observations can you make about the ratio of the sides and the areas?" 	<ul style="list-style-type: none"> Ratios of sides and perimeters are the same for similar figures. Areas are the square of the linear ratio.
<p>Homework</p>	<ul style="list-style-type: none"> EF-10 through EF-17. 		

Day 2 Enlarging and Reducing Figures

Connections

Prior Work	Current Big Idea	Future Work
<ul style="list-style-type: none"> Operations with integers. Ratio as equivalent fractions. 	<ul style="list-style-type: none"> Ratio between numbers. Ratio from a geometric perspective - enlarging and reducing simple lattice polygons. 	<ul style="list-style-type: none"> Solve equations involving proportions.

Lesson Process

Steps	Student Activity	Teacher Support	Comment/Evaluation
Launch 8-15 minutes	<ul style="list-style-type: none"> Use EF-7 and EF-8 to predict the ratio of the sides, perimeters, and area when a figure is enlarged three times. Add definitions to the toolkit. Complete EF-19 with teacher and class. 	<ul style="list-style-type: none"> Use EF-7 and EF-8 to predict the ratio of the sides, perimeters, and area when a figure is enlarged three times. Work and discuss EF-19 together with students. 	<ul style="list-style-type: none"> Will students be able to predict the ratio of the sides, perimeters, and area when a figure is enlarged or reduced?
Explore 20-25 minutes	<ul style="list-style-type: none"> Complete EF-20 through EF-22 with study team. 	<ul style="list-style-type: none"> Check the teams' diagrams as you circulate. Watch for students who compute perimeters by counting dots rather than segments. 	<ul style="list-style-type: none"> Probability is a ratio comparison.
Summarize	<ul style="list-style-type: none"> Share conclusions from EF-20 through EF-22. 	<ul style="list-style-type: none"> Verify prediction of ratio of the sides, perimeters, and area when figures are 	<ul style="list-style-type: none"> Figures need to be similar for predictions.

Steps	Student Activity	Teacher Support	Comment/Evaluation
8-10 minutes		reduced or enlarged.	
Homework	<ul style="list-style-type: none">• EF-24 through EF29.		

Day 3
Ratio of Perimeters and areas

Connections

Prior Work	Current Big Idea	Future Work
<ul style="list-style-type: none"> Operations with integers. Ratio as equivalent fractions. 	<ul style="list-style-type: none"> Ratio between numbers. Ratio from a geometric perspective - enlarging and reducing simple lattice polygons. 	<ul style="list-style-type: none"> Solve equations involving proportions.

Lesson Process

Steps	Student Activity	Teacher Support	Comment/Evaluation
<p>Launch</p> <p>5-8 minutes</p>	<ul style="list-style-type: none"> Participate in discussion on the Distributive Property of EF-25. 	<ul style="list-style-type: none"> Start with brief discussion of the Distributive Property from EF-25. 	
<p>Explore</p> <p>20-30 minutes</p>	<ul style="list-style-type: none"> Complete EF-30 through EF-35 with study team. 	<ul style="list-style-type: none"> Some students may need more concrete questions for EF-31, such as "If $N = 2$ or $N = 3$, we know the answer. What if $N = 10$? Or $N = 100$?" Let student decide on how to solve the equations in EF-33. 	<ul style="list-style-type: none"> A conjecture is just an educated guess. Do not ask student to justify why the area ratio is N^2.
<p>Summarize</p> <p>8-10 minutes</p>	<ul style="list-style-type: none"> Share conjectures with class. Add conjectures to toolkit. 	<ul style="list-style-type: none"> Discuss and share conjectures from EF-30 through EF-32. 	<ul style="list-style-type: none"> Check students' understanding of conjectures.

Steps	Student Activity	Teacher Support	Comment/Evaluation
Homework	EF-37 through EF-40, EF-42, EF-43.		

Day 4
Ratio of Perimeters and Areas

Connections

Prior Work	Current Big Idea	Future Work
<ul style="list-style-type: none"> • Ratio as equivalent fractions • Ratios of similar figures. 	<ul style="list-style-type: none"> • Examine the ratios of corresponding sides of similar triangles. 	<ul style="list-style-type: none"> • Solve equations involving proportions.

Lesson Process

Steps	Student Activity	Teacher Support	Comment/Evaluation
<p>Launch</p> <p>5-8 minutes</p>	<ul style="list-style-type: none"> • Work with teacher and class on EF-44 and EF-45. • Add definitions relating to similar triangles to toolkit. 	<ul style="list-style-type: none"> • Define conditions for similar triangles. • Give the format for stating the length of segments. • Suggest the students mark corresponding sides of similar triangle with the same color. 	<ul style="list-style-type: none"> • Students may need help with the convention that angles marked with the same symbol have equal measures.
<p>Explore</p> <p>20-25 minutes</p>	<ul style="list-style-type: none"> • Complete EF-45 through EF-47 with study team. • Add definition of 'proportion' to toolkit. 	<ul style="list-style-type: none"> • Ask students to summarize their finding from EF-46 in their own words. • The ratio of a pair of sides of one triangle is equal to the ratio of the corresponding pair of sides of a similar triangle. • Students should recognize the difference between parts (d) and (e) of EF-46. 	<ul style="list-style-type: none"> • Students should have an idea that ratio will work whether comparing sides within similar triangles or between similar triangles.

Steps	Student Activity	Teacher Support	Comment/Evaluation
<p>Summarize</p> <p>8-10 minutes</p>	<ul style="list-style-type: none"> Discuss and share conclusions for EF-47 with class. 	<ul style="list-style-type: none"> Students should recognize the difference between the ratio of corresponding pairs of sides of a similar triangle and the ratio between corresponding sides of similar triangles. 	<ul style="list-style-type: none"> Students should be able to use the equivalent ratios to write equations.
<p>Homework</p>	<p>EF-48 through EF-35.</p>		

Day 5 Ratio in Right Triangles

Connections

Prior Work	Current Big Idea	Future Work
<ul style="list-style-type: none"> Ratio as equivalent fractions. Ratio for pairs of corresponding sides of similar triangle. Vocabulary related to angles and triangles. 	<ul style="list-style-type: none"> Ratios of Similar triangles. 	<ul style="list-style-type: none"> Solve equations involving proportions.

Lesson Process

Steps	Student Activity	Teacher Support	Comment/Evaluation
Launch 5-8 minutes	<ul style="list-style-type: none"> Add definitions, right angle, right triangle, to toolkit. Read PZL-19. 	<ul style="list-style-type: none"> Read PZL-19. 	<ul style="list-style-type: none"> Focus on ratios of similar triangles through right triangles.
Explore 20-25 minutes	<ul style="list-style-type: none"> Complete EF-54 through EF-58 with study team. 	<ul style="list-style-type: none"> Students should recognize the difference between the ratio of corresponding pairs of sides of a similar triangle and the ratio between corresponding sides of similar triangles. 	
Summarize	<ul style="list-style-type: none"> Discuss and share solutions to EF-55 through EF-58 with class. 	<ul style="list-style-type: none"> Select study teams to present solutions to EF-55-EF-58. 	

Steps	Student Activity	Teacher Support	Comment/Evaluation
10-15 minutes			
Homework	EF-60 thorough EF-65.		

Day 6
Percent and Proportion
Connections

Prior Work	Current Big Idea	Future Work
<ul style="list-style-type: none"> Ratio as equivalent fractions. Ratio for pairs of corresponding sides of similar triangle. 	<ul style="list-style-type: none"> Ratios of Similar triangles. 	<ul style="list-style-type: none"> Solve equations involving proportions.

Lesson Process

Steps	Student Activity	Teacher Support	Comment/Evaluation
<p>Launch 5-8 minutes</p>	<ul style="list-style-type: none"> Complete EF-66 and EF-67 with teacher and class. 	<ul style="list-style-type: none"> Graph horizontal and vertical lines. Encourage students to use graph paper. Do not try to make this “easy” with memorized procedures. Let students generate their own algorithm for determining horizontal and vertical lines. 	<ul style="list-style-type: none"> EF-67 is the preliminary work for ideas that will be used in Unit 7 for the concept of the slope of a line.
<p>Explore 20-25 minutes</p>	<ul style="list-style-type: none"> Complete EF-68 through EF-73 with study team. 	<ul style="list-style-type: none"> Let students use strategies developed in Unit 4 to solve the equations in eF-68. 	<ul style="list-style-type: none"> EF-68 through EF-73 is basically review, practice, and consolidation of ideas.
<p>Summarize 8-10 minutes</p>	<ul style="list-style-type: none"> Discuss and share conclusions on EF-71 with class. 	<ul style="list-style-type: none"> Select study team to present and discuss EF-71 with class. 	

Steps	Student Activity	Teacher Support	Comment/Evaluation
Homework	EF-74 through EF-80.		

Day 7
Equivalent Ratios and Graphs

Connections

Prior Work	Current Big Idea	Future Work
<ul style="list-style-type: none"> Ratios in similar triangles. 	<ul style="list-style-type: none"> Tie together the algebraic and the geometric views of ratios by examining similar triangles formed by a line on a set of coordinate axes. Use a line and similar triangles as models for making predictions. 	<ul style="list-style-type: none"> Determine equations for specified lines.

Lesson Process

Steps	Student Activity	Teacher Support	Comment/Evaluation
<p>Launch</p> <p>10-15 minutes</p>	<ul style="list-style-type: none"> Complete demonstration problem with teacher. 	<ul style="list-style-type: none"> Demonstrate problem on page 27 of Unit 5 (Teacher Version). Ask students processing questions related to the use of similar triangles and proportions involved. 	<ul style="list-style-type: none"> Do not rush to define slope. Allow students to use their intuitive understanding of slope and their new knowledge of similar triangles to solve the problems.
<p>Explore</p> <p>20-25 minutes</p>	<ul style="list-style-type: none"> Complete EF-81 with teacher and class. Complete EF-82 through EF-84 with study team. 	<ul style="list-style-type: none"> Complete EF-81 with class. In part (a) and (b) of EF-81, encourage students to use the problem solving techniques of making a table and looking for a pattern to develop the equation. 	<ul style="list-style-type: none"> Guide students in using similar triangles to set up proportions.

Steps	Student Activity	Teacher Support	Comment/Evaluation
		<ul style="list-style-type: none"> • In part (g) of EF-81, guide students back to the definition of similar triangles. • Guide students in writing proportions to solve EF-83 and EF-84. • Students may need to try organizing the information in a table as shown in EF-81. 	
<p>Summarize</p> <p>8 – 10 minutes</p>	<ul style="list-style-type: none"> • Share conclusions for EF-82 with class. 	<ul style="list-style-type: none"> • Select study teams to share conclusions for EF-82. 	
<p>Homework</p>	<ul style="list-style-type: none"> • EF-86 through EF-91. 		

Day 8
Writing and Solving Equations Involving Ratios

Connections

Prior Work	Current Big Idea	Future Work
<ul style="list-style-type: none"> Ratios in similar triangles. 	<ul style="list-style-type: none"> Tie together the algebraic and the geometric views of ratios by examining similar triangles formed by a line on a set of coordinate axes. Use a line and similar triangles as models for making predictions. 	<ul style="list-style-type: none"> Determine equations for specified lines.

Lesson Process

Steps	Student Activity	Teacher Support	Comment/Evaluation
<p>Launch</p> <p>5-8 minutes</p>	<ul style="list-style-type: none"> Complete EF-93 and EF-94 with teacher and class. 	<ul style="list-style-type: none"> Complete EF-93 and EF-94 with class. 	<ul style="list-style-type: none"> In EF-93 instead of writing and solving equations with proportions ($\frac{1}{5} = \frac{x}{30-x}$), students could solve equations that involve ratios implicitly ($x + 5x = 30$).
<p>Explore</p> <p>20-25 minutes</p>	<ul style="list-style-type: none"> Complete EF-94 through EF-99 with study team. 	<ul style="list-style-type: none"> Encourage students to use the problem solving techniques used in Unit 4. Student may use Guess and Check to 	

Steps	Student Activity	Teacher Support	Comment/Evaluation
		write and solve the problems.	
Summarize 10-12 minutes	<ul style="list-style-type: none"> • Share conclusions from EF-94 through EF-99 with class. 	<ul style="list-style-type: none"> • Select study teams to share EF-94 through EF-99 with class. 	
Homework	<ul style="list-style-type: none"> • EF-101 through EF-107. 		

Day 9
Estimating Fish Population – A Simulation

Connections

Prior Work	Current Big Idea	Future Work
<ul style="list-style-type: none"> Solve equations involving proportions. 	<ul style="list-style-type: none"> Simulation of procedure used by Fish and Game scientists to estimate the number of fish in a lake at a given time. 	<ul style="list-style-type: none"> Simulations of real-world situations.

Lesson Process

Steps	Student Activity	Teacher Support	Comment/Evaluation
Launch 8-10 minutes	<ul style="list-style-type: none"> Read EF-108. 	<ul style="list-style-type: none"> Read page 33 of Unit 5 (Teacher Version). Select student to read the overview aloud to class. 	<ul style="list-style-type: none"> Expect to spend at least 40 minutes on this activity.
Explore 30-35 minutes	<ul style="list-style-type: none"> Complete EF-108 with study team. 	<ul style="list-style-type: none"> Guide students in completing EF-108. Part (f) contains an extension. Parts (g) and (h) are not optional. 	
Summarize 5 minutes	<ul style="list-style-type: none"> Post results of EF-108. 	<ul style="list-style-type: none"> Post results from eF-108 from study teams. 	
Homework	EF-109 through EF-115.		

Day 10 Summary

Connections

Prior Work	Current Big Idea	Future Work
<ul style="list-style-type: none"> Write and solve equations involving proportions. 	<ul style="list-style-type: none"> Review ratios from geometric, algebraic, and numeric perspectives. 	<ul style="list-style-type: none"> Write and solve linear equations.

Lesson Process

Steps	Student Activity	Teacher Support	Comment/Evaluation
Launch 10-15 minutes	<ul style="list-style-type: none"> Complete EF-116 with teacher and class. 	<ul style="list-style-type: none"> Generate a class list of “Big ideas” for Unit 5. Narrow the list down to roughly four items. 	
Explore 20-25 minutes	<ul style="list-style-type: none"> Complete EF-117 through EF-126 with study team. 	<ul style="list-style-type: none"> For EF-117, suggest drawing a picture of two triangles. For EF-118, suggest making a table and constructing graphs for each of the coins. 	
Summarize 5 minutes	<ul style="list-style-type: none"> Share completed EF-117 through EF-126 with class. 	<ul style="list-style-type: none"> Suggest students share or exchange completed problems with other study teams. 	

Steps	Student Activity	Teacher Support	Comment/Evaluation
Homework	<ul style="list-style-type: none">• Complete unfinished work from EF-117 through EF-126.		